Mastering Circuit Analysis: An innovative approach to a foundational sequence

William J. Leonard, C.V. Hollot, and William J. Gerace
University of Massachusetts Amherst, leonard@ecs.umass.edu, hollot@ecs.umass.edu, gerace@physics.umass.edu

Abstract - The Department of Electrical and Computer Engineering at the University of Massachusetts Amherst has dramatically changed how Circuit Analysis is learned. Combining a traditional lecture/recitation format with secure, online tests, we have raised our expectations for students and improved student performance, while increasing the fraction of students who succeed, especially underrepresented minorities and women. We call the instructional approach “Mastery”, because students continue to work on a topic until they earn a perfect score on the corresponding test. Further, it shows great promise for transforming undergraduate education, especially those courses that provide foundational skills and prepare students for future learning. The approach could have a positive impact on the retention rates of all students and even greater impact on those for underrepresented groups. In this paper, we describe the Mastery approach and its implementation over the past two years. We also show the current state of passing and completion rates as compared to those rates before Mastery was introduced. Finally, we discuss a range of issues, including the appropriateness of the Mastery approach for other courses.

Index Terms - Mastery learning, Circuit analysis, Retention rates, Preparation for future learning.

A HISTORY OF MASTERY LEARNING

The mastery approach to learning is not a new idea. For countless generations, apprentices have been learning skills from an expert, with well defined competency criteria to be called a “master”. As applied to the classroom, Block [1] and Horton [2] report that mastery was popular in American schools during the 1920s, but interest in it waned in the 1930s, and then the ideas reappeared in the 1950s and 1960s. John Carroll [3] is often credited with giving mastery its first theoretical underpinnings, and Benjamin Bloom [4],[5] is given preeminence for providing the most complete model of mastery learning, with classroom experience and numerous studies to inform and refine his model.

The philosophy that emerged is that with enough time, motivation, and feedback, everyone can learn any subject. The 8 common tenets of mastery are:

1. The subject matter should be broken down into well defined, well sequenced units.
2. Criterion for mastery should be explicit (usually 80% or 90%).
3. Diagnostics should be available.
4. Feedback should be provided.
5. Students should be allowed to work through units at their own pace.
6. Students should not be allowed to move on to more sophisticated units until they have mastered more basic units.
7. Supplemental activities should be used to remediate failure to master a unit.
8. A summative exam should be created for students to demonstrate overall mastery.

Personalized Systems of Instruction (PSI) [6] are often lumped together with mastery approaches, though there are some critical differences. As its name suggests, PSIs are usually done by individuals working alone with the teacher playing an advisory role, while mastery as developed by Bloom is usually implemented in a classroom setting with students helping each other and the teacher playing a much more central role, especially in delivering content. Also, the use of PSI is more common in college settings, while mastery is more common in elementary and secondary settings. Nonetheless, PSI shares many of the philosophical premises with mastery. Further, all 8 of the common tenets of mastery listed above apply to PSI.

Hundreds of studies of mastery have been done, mostly at the primary and secondary levels. At least five large-scale meta-analyses have been done, four of which show that mastery learning has numerous positive effects [7-10], including increased confidence, self-esteem, and interest in the subject [11]. The fifth [12] acknowledges the positive effects but suggests that most studies underemphasize the role of the teacher in the reported effects, because the teachers design both the instructional units and the performance criteria for demonstrating mastery. In engineering more than 30 years ago, mastery learning using PSI failed to show any retention of mastered material [13].

Mastery-based grading is not the same as mastery learning, but it has promoted learning in a few engineering courses. In particular, a number of instructors have used a system of grading in which students are allowed to retake assessments with no penalty [14-16]. In mastery-based grading, the course structure remains traditional, including paper-and-pencil tests. All report improvements in student learning and a reduction of fear and stress associated with being tested. Armacost and Pet-Armacost [14] applied the
technique in an operations research class, and students simply had the option of retaking or resubmitting any graded assessment until they were satisfied with their score or until the semester ended. Students liked the approach. Carpenter [15] applied the technique in civil engineering, and students were required to retry selected exam questions until they earned a perfect score. There were five questions in all, and by the fourth variation, all students had demonstrated mastery. Ray and Yates [16] developed the “A, B, or retry” policy; students who earned below 80% on midterm exams were required to retake them. Others had the option of retaking them for a higher score. The policy helped students take more responsibility for their own learning. The authors also report improved passing rates and retention. More than 10 years later, the policy is still being used, and graduation rates have improved [17]. The main drawback of all three examples is the burden of creating and hand-grading paper-and-pencil tests, which makes the techniques burdensome and impractical to scale up to all course material and/or large classes.

ENGINEERING CIRCUIT ANALYSIS AT THE UNIVERSITY OF MASSACHUSETTS

Circuit Analysis is a full-year, foundational sequence for undergraduate Electrical Engineering (EE) and Computer Systems Engineering (CSE) majors. It feeds into two strands of EE—Systems and Electronics—and it broadens the knowledge base for CSE. The sequence is required for all majors, and most take it in their second year.

Table I and Figure 1 show some statistics associated with the Circuits I-II sequence over the past six years. We focus on students who enrolled in Circuits I for the first time, in part because our long-term goal is to see students complete the sequence in one year and stay on track toward graduation and in part because students who have failed to complete the sequence in one year are much more likely to continue to fail, making the comparison overly biased toward Mastery if all enrolled students are included.

The grade of C is critical in both courses, because it is needed to advance to the next course or set of courses. Thus, in the vernacular of the students, “C or above” is passing. We will adopt this same definition. Further, we define one-year completion to be the state of passing both courses (or their equivalents) within one year of enrolling in the first course for the first time.

During the four academic years before implementation of Mastery, a cumulative average of 76% of students passed Circuits I the first time they enrolled in it, and nearly all of these students then enrolled in Circuits II the following semester. Almost 90% of them passed Circuits II, for a one-year completion rate of about 66%, i.e., about two-thirds of students who enrolled in Circuits I in the Fall semester earned at least C in both courses during the academic year, though the rate varies from about 60% in AY02-03 to about 70% in AY04-05 and AY05-06.

In its first two years of implementation, only 64% of Mastery students passed Circuits I, suggesting that the first half of the sequence is now harder than it was in the previous four years. However, with the addition of an interterm course in January, many of those who did not pass Circuits I during the Fall semester were able to learn the requisite material and fulfill the enrollment requirements for Circuits II. Thus, more than 80% of the original 125 students enrolled in Circuits II. Including those who passed a summer course in Circuits II in 2007, about 77% of students passed both courses within one year of entering Circuits I, which is notably higher than the 66% of students who finished the sequence in one year prior to implementation of the Mastery approach. After another summer course in 2008, we expect the Mastery completion rate to be even higher.

Minority students (African Americans, Hispanics, and women) have fared poorly with the conventional approach. (We don’t have a complete set of institutional data, so the results reported here are only approximate.) In the four years prior to Mastery, almost 60 minority students enrolled in Circuits I for the first time. Of these, the pass rate in Circuits I was about 65% and the (one-year) completion rate for the sequence was about 55%, both of which are about 15 percentage points lower than the rates for non-minorities (which are about 5% higher than the rates shown in Table I). By contrast, of the 20 students who have enrolled in Circuits I in Fall 2006 and 2007 with the Mastery approach, only 13 passed, but all 20 passed its equivalent and enrolled in Circuits II. Eighteen passed Circuits II, for a one-year completion rate of 90%.

TABLE I

| Statistics for Circuit Analysis I and II over the Past Six Academic Years. For Each Percentage, the Standard Error is Shown in Parentheses. Treating the Four “Conventional” Years As Equivalent to Each Other, Quantities Are Computed Using Cumulative Values (Labeled “4 AYs”), Making It Easier to Compare the Conventional Approach with the Mastery Approach. Likewise, the Two “Mastery” Years Are Combined and Labeled “2 AYs”. After Summer 2008, the Last Two Columns Will Change. Results Are Shown Graphically in Figure 1. |
|-------------------------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|
| (a) Number of students enrolled in Circuits I for the first time | Conventional Approach | Mastery Approach |
| F02-S03 | F03-S04 | F04-S05 | F05-S06 | 4 AYs | F06-S07 | F07-S08 | 2 AYs |
| 104 | 110 | 65 | 74 | 353 | 63 | 62 | 125 |
| (b) Percent (+/-) of students in (a) who earned at least C in Circuits I | 74 (4) | 75 (4) | 82 (5) | 76 (5) | 76 (2) | 65 (6) | 63 (6) | 64 (4) |
| (c) Percent (+/-) of students in (a) who enrolled in Circuits II the next semester | 71 (4) | 73 (4) | 82 (5) | 76 (5) | 75 (2) | 83 (5) | 82 (5) | 82 (3) |
| (d) Percent (+/-) of students in (a) who passed both Circuits I and II within one year | 65 (5) | 59 (5) | 72 (6) | 72 (5) | 66 (3) | 79 (5) | 74 (5) | 77 (4) |

978-1-4244-1970-8/08/$25.00 ©2008 IEEE  
October 22 – 25, 2008, Saratoga Springs, NY  
38th ASEE/IEEE Frontiers in Education Conference  
F2H-4
These statistics strongly suggest that the Mastery approach helps all students, but it helps minority students even more than other students.

**Motivation and Implementation**

The current project started in July 2005. The circuit analysis sequence had recently ended, and the professors involved were concerned that a number of students had earned Bs and Cs without having ever answered a single exam question 100% correctly. They had earned passing grades in part because of partial credit on exams and in part because so much weight was given to lab reports and written homework. We were concerned that these students might not be prepared for upper division courses. It was tempting to make exams worth more, but we felt that raising the stakes would make matters worse instead of better. We wondered if it might be possible for students to completely master some fraction of the material, perhaps 75%, rather than learn 60% of all the material. If the testing were done online with a large database of questions and contexts, students could retake exams until they earned a suitable score, thus lowering the stakes on each exam. After a year of design, development, field-testing, piloting, and revision, the first on-sequence cohort of students took Circuits I in September 2006.

*I. Course structure and design*

The Mastery approach has many facets that distinguish it from other instructional modes. The core of the approach is a set of 16 modules (per semester). First, the subject matter is divided into 12 Basic modules that stress development of useful skills and key concepts. These roughly correspond to what would be covered in one week of instruction. Then, three Intermediate modules integrate ideas and skills covered in related groups of Basic modules, roughly corresponding to the content and level of midterm exams. That is, these modules emphasize the decision-making, knowledge-structuring, and higher level thinking skills associated with expert-like problem solving. Finally, a Cumulative module integrates ideas and techniques from the entire course, roughly corresponding to a final exam, stressing even higher level thinking and analysis skills.

Modules are carefully sequenced to reflect the structure of circuit analysis. During the Fall semester, we cover concepts and techniques useful for solving dc (direct current) networks. Fundamental ideas are stressed over and over as the material proceeds from analysis of purely resistive networks to those with resistors combined with either capacitors or inductors to those with resistors, capacitors, and inductors. During the Spring, we move to ac (alternating current) networks, again with modules building on earlier material. In fact, the second course has relatively few new ideas, and a solid foundation from the first course makes it an easier course than most.

Each course retains a conventional format of three 50-minute lectures and one 75-minute recitation per week, so it is primarily the assessment criteria that have been changed. The modules provide meaningful motivation, since students generally know what they need to learn and what they don’t know yet, making them much more active participants in the learning process.

Both courses have a project component in which students, often working in pairs or small groups, learn useful computer software tools in the context of circuit analysis (PSpice, Excel, and MATLAB). The second semester course also has a lab component, during which students work in pairs to model, analyze, build, and take measurements of real circuits.

*II. Module structure, design, and logistics*

Each module has 10 questions, and a perfect score is needed to earn mastery. When a student fails to master a module, he must retake the entire test, including those questions he answered correctly before. The tests are electronic and secure, with literally thousands of combinations possible, so each one is unique and cheating is not possible. (Students must go to a monitored computer classroom on campus to make Mastery attempts.) When a student makes another attempt at a module, he gets a completely new test; not only are the values for all parameters different, the circuits are different as well and often the questions are different too.

Online Practice modules are available but completely optional; students earn no course credit for doing them. After each attempt at a question, they are given feedback in the form of the correct answer and whether or not their answer is correct. Sometimes they are also given some written feedback associated with the question, such as common pitfalls or useful ways of thinking about the context. Students are allowed to redo each question as many times as they wish. Each new attempt recasts the question with a new set of values for variables and a new circuit, thus allowing students to continue until they are able to answer the question correctly multiple times. However, the database of practice contexts is different from the Mastery database. Students are encouraged to practice for two hours or so before making their first Mastery attempt. They can ask questions in the weekly recitation, during office hours, or by
email. Many students work in pairs or small groups when practicing.

Online Mastery modules are attempted at a monitored, secure site when students decide they are ready to do so. Students work through the entire exam before submitting it to be scored, though they can skip from question to question and change answers if desired. For Basic modules, each test is designed to take 20 to 30 minutes to complete; if a student has truly mastered the material. The tests have a one-hour time limit to give students sufficient time to find and fix mistakes. Intermediate and Cumulative modules have longer time limits. One of three secure sites is open for about 25 hours per week.

III. Grading criteria and class management

Students’ grades are based predominantly on which modules they master before the end of the 16-week term (i.e., 14 weeks of classes and about 2 weeks of final exams). To earn a C, students must master 10 modules, including one Intermediate module. In other words, students must master 9 of 12 Basic modules, and then master one higher level module. (Students also must earn at least 175 out of 200 points on labs and computer projects.) Once a student has earned a C, the actual grade is determined by how many grade increments, e.g. C to C+, the student has earned. For instance, mastering an additional module earns one grade increment. Earning 15 “Bonus” points also earns one grade increment, which includes the sum of best scores on unmastered modules and earning at least 9 points on a Basic module before its “Bonus” date.

Many of the grading criteria are designed to assist with class management. For example, in the first year of implementation (AY06-07), there was no partial credit awarded for unmastered modules, and this created an enormous amount of frustration among the students. Bonus points were implemented in the second year as a way of reducing this frustration, and we decided that 15 points earned on unmastered modules was equivalent to mastering a single module, thus earning a grade increment.

Also, students are not accustomed to learning using the Mastery approach, and the absence of any meaningful due dates can make it difficult for them to manage their time. When a student is taking another four or five courses with due dates at the same time as Circuits I or II, it can be hard to find the internal motivation to work on Practice exercises at home or make Mastery attempts at the secure site. Bonus dates help students manage their time better. For each Basic module, three of eight attempts are due a couple of weeks after the material has been covered in lecture. If a student earns 9 or more points on any of these attempts, he earns one Bonus point, which can be combined with other Bonus points at the end of the semester to earn a grade increment. Even though one point does not seem like much, it is enough to give most students the incentive to at least attempt a module earlier rather than later. Most realize that even one point can make the difference between one grade and the next.

IV. Student performance, grades, progress, and attitudes

Space limitations prevent an in-depth description or analysis of how individual students cope with the Mastery approach to learning, so a lengthy summary will need to suffice. Although the pass rate is relatively low in Circuits I (64%), given the new standards for earning course credit—perfect scores on a multitude of tests—performance is better than some might expect. In fact, the resulting distribution of grades in Figure 2 is far from the bell-shaped curve we have come to expect. The most dramatic feature of this distribution is perhaps the increasing number of students as the grade increases above C, with nearly 20% of all students earning an A. Another dramatic feature is the apparent barrier between C and C–. This is caused in part by the minimum requirement for a C and in part by Bonus points leading to grade increments for students who fulfilled the requirements for a C.

One virtue of the online modules is that they can be used as the core of interterm courses covering the same content. Thus, the department now offers two self-study courses that fulfill the same curricular requirements as Circuits I and Circuits II. Unlike many interterm courses, which are easier than their Fall or Spring counterparts, these courses are just as difficult to pass, because the grading criteria are the same. Also, because the modules are online, students can take the course without being on campus. (To ensure individual accountability, selected modules must be mastered at a secure site on campus when they visit during interterm or return at the end of the interterm.) These courses are useful for students who have already taken Circuits I or II, but did not quite meet the minimum requirements for a C. For instance, they might have attended lectures and recitations during the semester and have mastered 5 to 10 modules, but they have run out of time and did not master enough modules to pass the course. A high
The pattern of student performance in Circuits II is very similar to that in Circuits I (not shown), with the number of students earning each grade increasing as the grade increases. One difference is that there are relatively few students below C, i.e., the pass rate for students enrolled in Circuits II is about 90%.

Students’ opinions and views of mastery varied widely and evolved during the year. At the beginning of the year, many students were skeptical, in part because they did not fully understand what Mastery would be like. Most students were concerned about the requirement for perfection, since getting even one question wrong means they will have to make another attempt. By the middle of the first semester, there is quite a bit of frustration, mostly due to a perceived lack of progress. By the end of the first semester, a survey showed that stress was still present, but most students realized that the approach was working. Circuits II proceeded much more smoothly than Circuits I, with much less stress and frustration. By the end of the sequence, the entire class has become convinced that Mastery works.

**DISCUSSION**

The Mastery approach as implemented in Circuit Analysis at the University of Massachusetts Amherst shares many features with the common tenets of Mastery Learning as developed and studied in the 1960s and 1970s. There are some critical differences, though, due in part to the college context and in part to our own emphasis on developing analysis skills and expertise.

The Mastery approach also shares some features with Mastery grading in that a traditional lecture/recitation format is retained and the Mastery modules primarily impact grading, with students retaking tests until they earn a perfect score. We would argue however that the Mastery approach is not Mastery grading for the following reasons: In Mastery grading, traditional assessments are used and the “mastery” portion is relatively minor; the mode by which students are learning is conventional and students simply retake all or part of the assessments. In our approach, the use of Mastery modules permeates a very large fraction of what students are doing every day; yes, they attend 150 minutes of lectures and 75 minutes of discussion per week, but it is the Practice exercises and Mastery attempts that they are focused on most of the time. The modules simultaneously provide the motivation to learn, the formative feedback useful for making decisions about learning, the context of almost all discussions, and the criteria for grading.

I. Coverage vs. preparation for future learning

Students are very aware of “filter” courses like circuit analysis. They know that certain courses have a high failure rate, and they can only hope that they are in the top 75-80%. Most students will take those odds, perhaps because they do not know how to predict where they are in a class. As professors, we have grown accustomed to large numbers of students who will need to spend another year in school or who will need to change majors because they have failed to pass one of these courses. At the same time, we have also become increasingly dissatisfied with student performance. It seems sometimes that even though large percentages of students are failing, the ones who pass do not quite seem to know what those in previous generations used to know. On the one hand, it is tempting to make the foundational courses easier in order to allow more students to proceed to upper division courses, but on the other hand, that would immediately make our current dissatisfaction even worse.

The Mastery approach as described here appears to be harder than conventional approaches, with an even lower percentage of students passing Circuits I. We anticipate that students will be able to remember more of what they have learned, though it is likely that there will be units for which they know little or nothing, perhaps because they did not have time at the end of the semester. In other words, we have made a conscious decision to emphasize mastery of some of the course material over coverage of all. In particular, students can earn a C by mastering nine of 12 Basic modules. They literally can pass the course without ever attempting three of the Basic modules. In practice, only a few students are in this category. Nearly everyone masters 11 of 12 modules, and like most courses, the material at the end of the course is not necessarily intended to be mastered. In other words, at least for the typical B or C student, we are trading shallow coverage of all topics for more in-depth coverage of most topics. Like most innovations, a students merely adapt, while learning the same material.

Thus, the data suggest that even though the completion rates are higher than with conventional approaches, the course is actually harder, so we can attribute the Mastery approach directly with simultaneously raising standards and improving success. Further, just as enhanced success in Circuits II is likely to be due to students having mastered ideas from Circuits I, we anticipate that students who have learned circuit analysis using the Mastery approach will be better prepared for upper division courses.

II. Retention

The current enrollment trends for engineering in general and electrical and computer engineering (ECE) in particular are all negative. The National Center for Educational Statistics reports that nationwide the number of students entering colleges and universities each year is rising [18]. Unfortunately, the number of students entering engineering each year is staying roughly constant and the number choosing ECE is dropping [19]. The trends for minorities are even worse than those for the general population [20].

The Mastery approach has dramatically improved one-year completion rates, which have immediate consequences for retention. Whereas before Mastery about 66% of students finished the one-year sequence and were allowed to take upper division courses with their classmates, with...
Mastery nearly 80% of students can do so. It turns out that most of this effect is due to the performance of minority students, who have gone from a completion rate of around 55% before Mastery to 90% afterwards.

III. Appropriateness for other courses

We expect engineers to get problems right, at least computationally. We want them to analyze every situation they encounter, write down valid equations, and compute the correct answer every time. Somehow, with partial credit on exams and excessive credit for homework, it seems as though we have created a generation of engineers who do not think it is important to get the right answer. Every discipline has a set of “foundational” courses—courses that provide the fundamental skills needed to learn upper level material. For all engineers, calculus and differential equations are foundational, and for many, physics and/or chemistry are foundational. For mechanical, civil, and chemical engineers, the foundational courses are statics, probability and statistics, and/or thermodynamics. Look at the sophomore level courses for any engineering major, and you will find candidates for foundational courses suitable for the Mastery approach. All that is required is that there are well defined skills and techniques to be mastered and problems to be solved.

Some courses might not be suitable for the Mastery approach. For instance, an information-heavy course with an emphasis on memorization might not work as well as problem-solving courses. Also, courses with an emphasis on programming or design might not fit the approach, since it might be too challenging to create the online modules.

CONCLUSIONS

A Mastery approach has been designed, developed, field-tested, piloted, and used to teach a two-semester, second-year sequence in circuit analysis. Its structure corresponds to the 8 common tenets of Mastery Learning. It has been implemented twice (AY06-07 and AY07-08) with 125 students. Each student’s grade is determined predominantly by how many secure, online modules he or she masters. Students make Mastery attempts only when they are ready. A large database of contexts and questions makes it feasible to let students continue to make attempts until they have achieved mastery by earning a perfect score. The online modules also serve as the basis for two interterm courses that help maintain high standards for the sequence while giving students alternative paths to success. Completion rates for the sequence are higher than rates before Mastery was implemented, especially for underrepresented groups.

We believe that the Mastery approach provides the motivation students need to learn critical skills and techniques. We also believe that the Mastery approach applied to appropriate foundational courses could transform engineering instruction and give many more students the knowledge, skills, and attitudes needed to become successful academically and ultimately successful professionally.

ACKNOWLEDGMENT

This work is supported in part by National Science Foundation grant ESI-0456124.

REFERENCES