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Sub-millimeter Positioning and Sensing for Short-Range Gravity Tests

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Abstract

With Einstein's theory of General Relativity accurately describing the fundamental force of gravity and the Standard Model of quantum mechanics describing the strong, weak and electromagnetic forces, there has yet to be a theory that unifies the two in a consistent manner. Attempts to do so have led to versions of String Theory that predict additional dimensions that would affect the gravitational inverse-square law (ISL) at sub-millimeter levels. It also predicts the existence of unobserved subatomic particles, which if exist, would cause a violation Weak Equivalence Principal (WEP) at short distances. The weakness of the gravitational force makes observation very difficult at small scales. Testing gravitational forces requires highly isolated experimental systems and precise measurement and control instrumentation. At the Humboldt State University (HSU) Gravitational Research Laboratory, we are developing an experiment intended to observe gravitational interactions at sub-millimeter distances in attempts to observe possible deviations from the gravitational inverse-square law and violations of the Weak Equivalence Principal. The experiment optically measures the twist of a novel parallel-plate torsion pendulum isolated in a vacuum chamber as an attractor mass is oscillated nearby, providing time-varying gravitational torque. In order to position and oscillate the attractor mass, we needed to implement precise positioning and sensing instrumentation inside the vacuum chamber. In addition to providing a brief overview of the background and objectives of the experiment, this paper discusses the implementation and role of a non-contact, capacitive position sensor in conjunction with a linear, open loop piezo-electric translation stage used to position an attractor mass within 50 microns of the torsion pendulum. The paper will focus on the techniques and theory used to develop a capacitive position sensor with sufficient accuracy required for the experiment, as well as the use of PID control theory as a means to precisely control the translation stage.

Keywords: Gravity, Inverse-square Law, Capacitive Sensing, Sub-millimeter Positioning, Control Theory

1. Introduction and Motivation:

Even though gravity is arguably the most apparent fundamental force in our day-to-day lives, many aspects of it still remain a mystery. Much of this mystery is a result of how weak the gravitational force is, making it extremely difficult to make precise measurements of its effects at small scales. For comparison, gravity is 10^{36} times weaker than the electromagnetic force. Additionally, attempts to create a "theory of everything," incorporating Einstein's theory of General Relativity and the Standard Model have given rise to a number of possible theories that predict unobserved gravitational behaviors on a small scale. Certain flavors of String Theory[1], such as ones that incorporate the effects of a fat graviton, predict that gravitational forces between two objects would decrease at a short enough distance scale. Other versions predict that gravitational forces may increase at these scales due to extra spatial dimensions[2-3]. Attempts to explain the Dark Energy phenomenon has led to theories predicting that gravity effectively turns off on scales as much as 0.1mm[4].

One way to model deviations from our classical understanding of gravity is to use a Yukawa addition[5] to the well-known equation for a gravitational potential between two objects of mass m_1 and m_2 separated by a distance r

$$V(r)=\frac{-Gm_1m_2}{r} \quad (1)$$

With the Yukawa addition, equation (1) becomes,

$$V(r)=\frac{-Gm_1m_2}{r}(1+\alpha e^{\frac{-r}{\lambda}}) \quad (2)$$

where α is a dimensionless constant corresponding to the strength of the deviation from known gravitational forces and λ corresponds to the separation distance between the two objects at which the deviation is occurring. Plotting this in α-λ parameter space alongside past experimental results[4, 6, 7] in Figure 1, we can easily identify regions of separation scales that have yet to be tested to confirm or refute classical Newtonian mechanics.

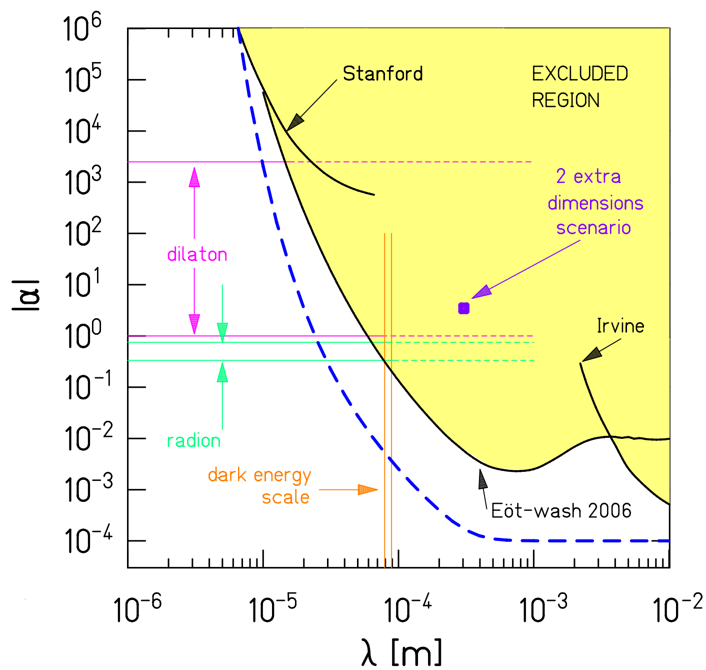

Figure 1. Yukawa parameter space.

Figure 1. Parameter space previously investigated at the 95% confidence level is represented by the yellow region. HSU's gravitational tests aim to investigate the region between the dotted blue line and past results. The regions labeled dilaton and radion represent length scales at which these unobserved particles are predicted to contribute to short range forces[8]. Also shown is the dark energy scale. The strength of the deviation from classical Newtonian mechanics corresponds to α at a separation distance corresponding to λ.

2. Humboldt State University Short Range Gravitational Tests:

2.1 experimental technique:

An experiment to observe the behavior of short-range gravitational interactions between objects separated down to 20 microns and below is currently in development at the Humboldt State University Gravitational Research Laboratory. The experiment aims to observe Newtonian gravitational interactions at these length scales to either confirm that classical Newtonian mechanics still applies, or, more excitingly, observe possible deviations from the predicted behavior providing information on the separation limits of classical theory as well as a magnitude of these deviations which can provide hints at possible sources of these deviations.

The experiment utilizes a stepped torsion pendulum and an attractor mass positioned within proximity of each other as seen in Figure 2. The relative size of the attractor mass is such that is appears as an effectively infinite planar mass to the torsion pendulum. The stepped geometry of the pendulum results in one side of the pendulum residing closer to the attractor mass than the other side. By modulating the separation distance between the plate and the pendulum, *s*, at an angular drive frequency ω, we will vary the gravitational force between the attractor mass and the pendulum. If there is any deviation from classical theory at the sub-millimeter level, the gravitational force between the pendulum and the attractor mass will vary between the two steps resulting in a torque on the pendulum causing it to twist. If Newtonian mechanics hold at this scale, the gravitational force from the effectively infinite plane will be equal on both steps of the pendulum and a torque is not applied. Measuring the twist angle of the torsion pendulum allows for the observed interaction to be compared with the predictions of Newtonian gravity.

2.2 experimental setup:

Due to the weak nature of gravitational forces, this system requires a high degree of isolation from environmental noise. The 10 g torsion pendulum is suspended using a 20 μm-diameter tungsten fiber, which is mounted from a magnetic damper that reduces swing and bounce oscillations. The pendulum is housed in a vacuum chamber pumped down to 10^{-6} Torr. This chamber is located in a temperature regulated room in a basement to further avoid unwanted vibrations or thermal expansions or contractions of the tungsten fiber. Using a torsion pendulum negates the effects of terrestrial gravity. Additionally, a 20 μm-thick sheet of BeCu foil will reside between the pendulum and attractor mass to reduce any electrostatic interactions between the two. The twist of the pendulum is measured optically with a laser reflected off of the back of the torsion pendulum. The reflection is then routed into a position-sensitive detector (PSD), which provides us with the position of the reflection upon the PSD. With this information and the systems geometry, the twist angle of the pendulum can be calculated. In order to reduce unwanted optical noise and unwanted modes of oscillation, data is collected from the optics through two SR830 lock-in amplifiers. All data is read and recorded using a National Instruments DAQ card and a custom data collection program created in LabVIEW. The attractor mass will be positioned with an open loop piezo electric motor controlled with LabVIEW utilizing a feedback loop that determines the appropriate control signal based off of a position sensor. The following sections will discuss the design decisions and development process of the position sensing system and control signal generation for use with this motor.

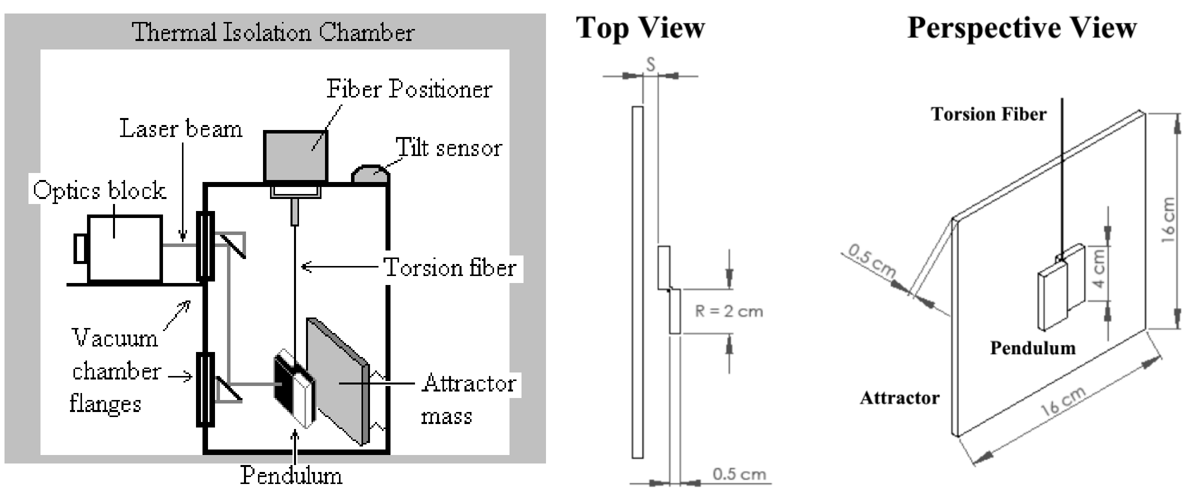

Figure 2. Experimental setup.

3. Attractor Mass Positioning:

The experiment requires that the attractor mass be positioned repeatably and accurately within ~1μm as well as having a system in place to provide information about its current position. Additional functionality would allow for direct distance measurements from the attractor mass to other surfaces or objects. The system must also be able to reach a position without damaging the delicate sheet of BeCu between the attractor mass and the pendulum, or disturbing the pendulum itself.

The piezo electric motor is a Physik Instrumente (PI) M-662.4V0 vacuum safe linear translation stage that is driven by accompanying PI C-185 Analog drive electronics. This motor's minimum incremental step size is rated at ~.05μm, well within our requirements. The motor is controlled using an analog voltage between -10 and 10 volts, which determines the direction and velocity of the motor. From initial tests of the motor's characteristics, it has been observed that the minimum reliable velocity is quite high (~1-3cm/s) and that the velocity varies unpredictably with position even at a constant control voltage, possibly due to the motors internal mechanisms. It is clear from these characteristics that some form of feedback loop that determines the motor's control signal based on information from a motion encoder or position sensor is needed.

Two different techniques have been attempted in an effort to develop a sensing and control system. The first method attempted, optical interferometry, provided the ability to take relative distance measurements with a resolution of a few hundred nanometers at a time. It presented a number of drawbacks and limitations however, and capacitive distance sensing was ultimately pursued due to its ability to make absolute distance measurements.

3.1 interferometry:

Due to equipment availability and familiarity, an attempt at designing and constructing a Michelson interferometer for motion encoding of the piezo electric motor was made. In this system, a Michelson style interferometer was constructed on a hydraulic dampened optics table using standard optical components normally used in educational experiments and a 500mW HeNe (650nm) laser. The piezo-electric motor was incorporated into one of the arms of the interferometer allowing the distance between the beam splitter and the mirror mounted onto the piezo electric motor to be modified. The interference pattern generated by the interferometer was then read into a LabVIEW program using a phototransistor. The goal of the system was to record the number of fringes that passed over the photo transistor as the distance in the variable arm of the interferometer was changed with either the piezo electric motor or the linear micrometer stage that the motor and the mirror were mounted too. With this information we would be able to calibrate the system to some known position, such as the motor stop, and then calculate the change in distance based on the number of fringes that pass by as the position of the motor is varied[9].

This arrangement ultimately failed to function acceptably, and would present considerable challenges when it came time to incorporate this position sensing system into our vacuum chamber due to its complexity and sensitivity to interference and disturbances. While the optics were able to function acceptably when the length of the arm was modified using the micrometer, which allowed for slower changes in distance, the characteristics of the piezo electric motor made it difficult to reliably read the fringe pattern due to its high minimum velocity. It was also decided that this method would not meet our requirements because it only provided relative change in position information, not allowing for direct distance measurements that are possible with a capacitive system.

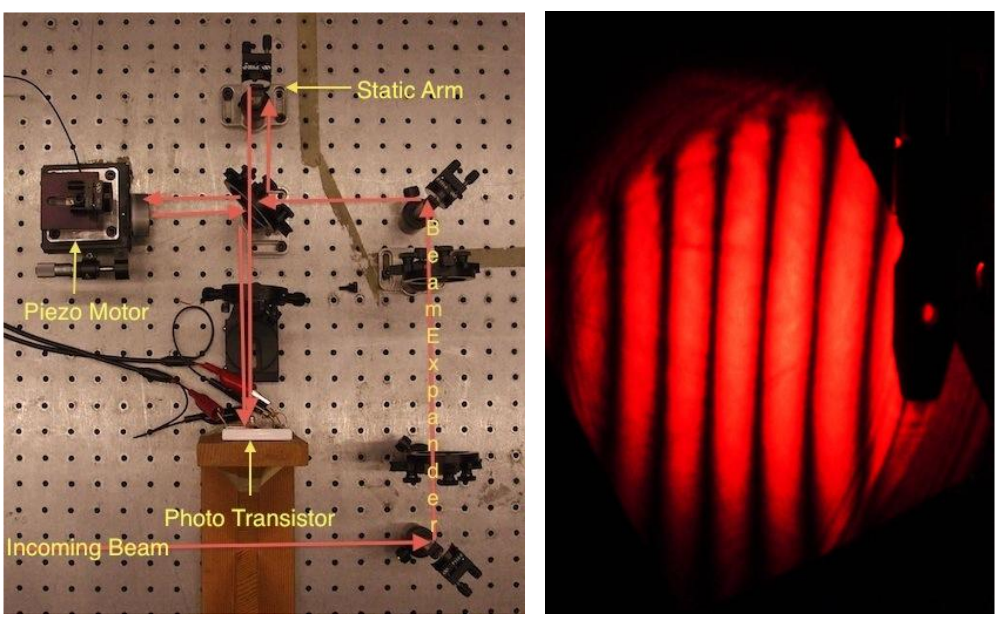

Figure 3. (Left)Interferometer-based motion encoder prototype, (Right) Sample Interference Pattern

3.2 capacitive motion encoding:

Commercial motion encoders and position sensors that are used in conjunction with piezo electric motors generally utilize some form of capacitive sensing[10]. Capacitive sensors offer non-contact, absolute displacement measurements between two objects with components that are generally smaller and simpler than those used with interferometry. A working prototype of a capacitive displacement sensor based feedback loop to control the piezo motor is currently completed, and efforts to design an improved system to address the prototypes weaknesses are currently underway.

3.2.1 two plate capacitive motion encoder prototype:

The working prototype utilizes a simple high pass filter circuit which can be seen on the right side of Figure 4. The capacitor is constructed using two crude copper plates, one that is mounted on the motor and another that is statically mounted parallel to the other. As the distance varies between the two plates as the motor moves, the capacitance between the two plates varies as approximately[11],

$$C = \varepsilon_0 \varepsilon_r \frac{A}{d} \tag{3}$$

where A is the overlapping area of the two plates, d is the distance between the two plates, and ε_0 and ε_r are the permittivity of free space and dielectric constants (which is ~1 for air) respectively. The system is excited using a function generator running a 15Mhz, 10Vpp square wave with a resistor value of 100 Ohms. The area of the plates used is ~8 cm^2. This configuration was able to produce discrete voltage values for a capacitor plate displacement range of ~1 cm. Larger ranges resulted in smaller voltage changes which were harder to differentiate to correspond to specific displacements. Fitting this data to the expected the values of V_{out} at a given displacement would be the next step to actually derive a way to take distance measurements, however due to the crude construction of this prototype, the stability of V_{out} was highly sensitive to external interference such as bodies moving throughout the room and misalignment of the plates. It was decided to construct an improved and more precise system instead of continuing with the current version. Even though our measured V_{out} was never matched to a distance scale, it was stable enough to construct a simple, logic based feedback loop that was able to position the motor to match V_{out} to a given set point voltage value. This system was capable of making quick and accurate movements in steps of millimeters, and it appeared to track accurately at sub-millimeter scales. A sample screen shot of the control software used in this prototype is featured in Figure 4.

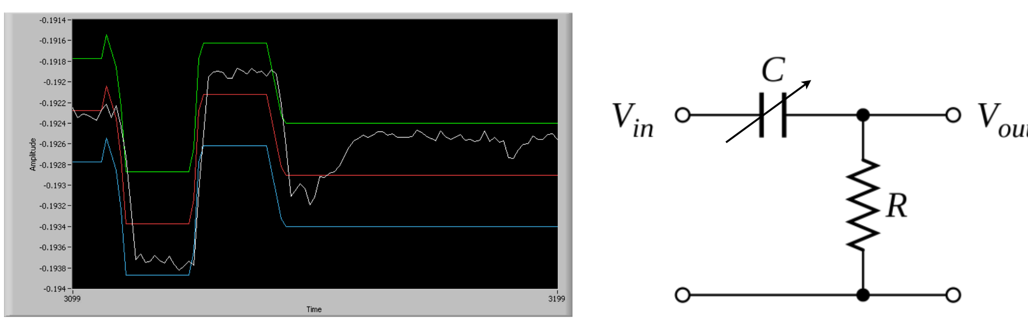

Figure 4. Current prototype of the capacitive sensor feedback loop for controlling motor position

Figure 4. The figure on the left is a screen shot of the control software as the set point for the position of the motor changes. The white line represents the voltage out of the circuit on the right. The variable capacitor in this prototype consists of two crude parallel plates exited with a function generator. The red line represents the set point the system attempts to match. The green and blue lines represent the tolerance of the system set by the user. When the white line resides within the region between the blue and green lines, the motor is turned off. If the white line is outside this region, the motor is turned on and moved in the correct direction until it resides within the tolerance region. The motion on this graph represents movements of ~1mm, however decreasing the tolerance allowed for more accurate motions.

3.2.3 future three plate motion encoder designs:

The two-plate prototype discussed above suffered from a limited range and stability issues. A number of things can be done to improve the range, such as increase the size of the capacitor plates, and decrease the electrical noise in the system. The stability issues were mostly the result of lack of adequate shielding around the system as well as the type of circuit used to measure the capacitance. For constructing a displacement based capacitive micrometer, it is recommended[12] that a three plate over-lap, under-lap plate configuration and a bridge type circuit to increase the range of linearity of the voltage out versus position of the sensor (See Figure. 7). The combination of these elements makes the system less susceptible to unwanted behavior due to plate misalignment. Additionally, the precision machining of the plates and their mounts should address much of the instability that was experienced with the current, crudely assembled prototype. The current plan is to use the circuit pictured in Figure 5. The proposed circuit would output a voltage based on the following equation[12],

$$E_0 = V\frac{C1 - C2}{Cf} = V\frac{2x}{d_0^2 - x^2}\frac{C_0}{Cf} \qquad (4)$$

where E_0 represents the output voltage of the circuit, V is the voltage output of the current generator, $C1$ and $C2$ represents the capacitance between the movable middle plate and the two end plates, Cf is a capacitor that is chosen to meet the requirements of the system, C_0 is the average capacitance of $C1$ and $C2$, d_0 is the distance between the two outer plates, and x is the position of the movable middle plate.

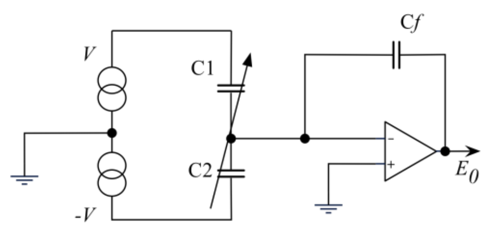

Figure 5. Proposed circuit for three plate design.[12]

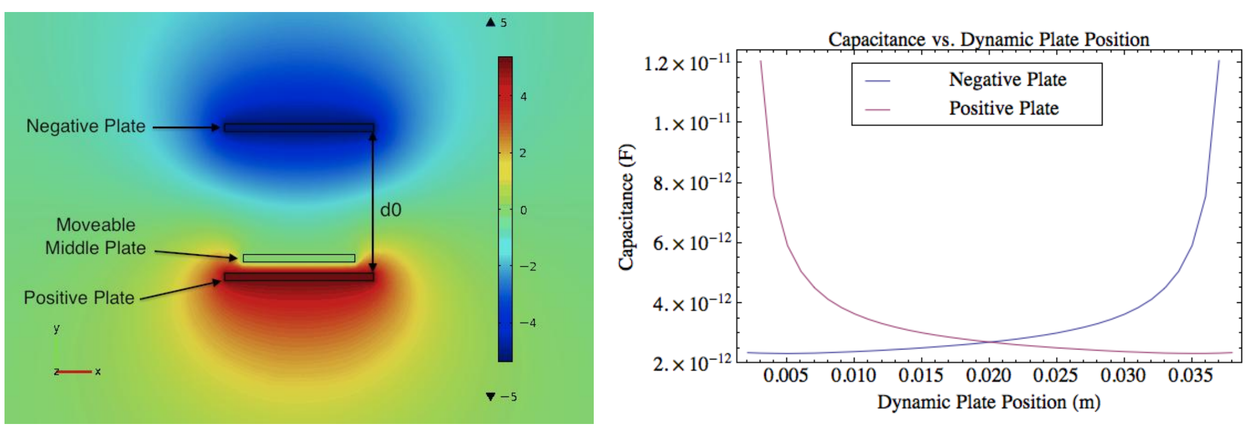

Figure 6. Simulated Three Plate Capacitive Distance Sensor Results

Figure 6. Finite element analysis is used to simulate the current design plan for the next version of the capacitive distance sensor. The plot on the left gives us an idea of the electric potential in space between the three plates. The right plot gives us theoretical values of the capacitance between the movable center plate and the static positive and negative plates as a function of center plate position.

We get an even better theoretical value for capacitance than what equation (3) predicts by modeling the full geometry of our proposed design using finite element analysis. While the final parts are still being developed, a simulation made up of geometric primitives was used to generate theoretical values for capacitance between the end plates and the middle plate based on the position of the middle plate. This is plotted in Figure 6. This simulation will eventually be expanded to incorporate the final geometry of the parts used in our actual system and will be used to match the output voltages of our system to position values.

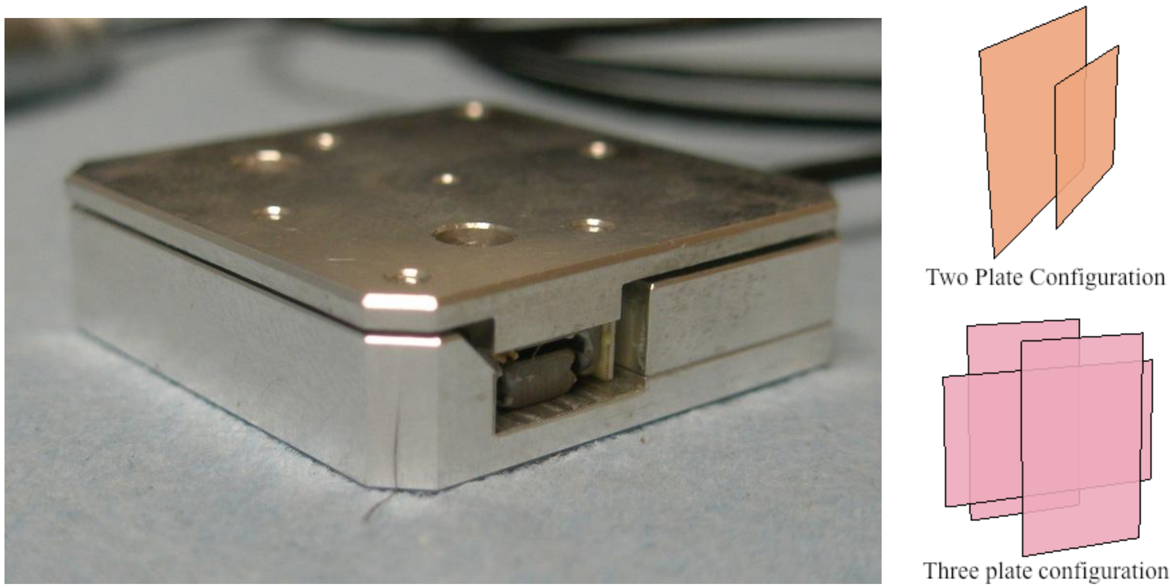

Figure 7. (Left) Image of the piezo electric motor stage, (Right) Two and three plate capacitor configuration

3.3 control theory:

In addition to the three-plate design, the use of Proportional, Integral, Differential (PID) control signals is predicted to improve the reliability of our feedback loop. Based on early attempts to implement PID control signals with the crude two-plate capacitive sensor design, a gain-scheduled, proportional and integral control signal is expected to meet the requirements of the project. Due to the non-linearity of the voltage vs. motor movement however, differential may still be a useful addition to determine the correct, instantaneous output voltage to send to the motor.

5. Conclusion:

Capacitive position sensors appear to be well suited for creating feedback loops for controlling open loop piezo electric motors. Attempts with interferometry were unsuccessful. In contrast, even crudely constructed capacitive sensor based prototypes were successful at creating a control system. While a modified displacement-based capacitive motion encoder appears to be able to meet the requirements of this project, it may ultimately attenuate the usable range of the motor due to its short range. Adding a secondary capacitive motion encoder could be a possible addition to detect a coarse position enabling use of the full range of the motor and may be perused in the future. Additionally, it is possible that direct measurements of the capacitance between the attractor mass and the BeCu sheet will be possible, which would enable direct measurements of the displacement between the two. This would likely have an extremely limited range and will likely require the three plate capacitive sensor or coarse capacitive motion encoder when the motor is positioned outside of its range. Once the motor and control system is completed, it will be mounted inside the vacuum chamber and initial short range gravitational tests incorporating the attractor mass can begin.

6. Acknowledgements:
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