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Experimental Short-Range Gravitational Tests of the Weak Equivalence Principle and the Inverse-Square Law Using Novel Torsion Pendulums

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Abstract

Of the four known fundamental forces, the force of gravity remains the only of these that is currently not well understood. While the Standard Model of quantum mechanics successfully describes the other three fundamental interactions, Einstein's theory of gravitation, General Relativity (GR), is fundamentally inconsistent with this model. A central feature of GR is the Weak Equivalence Principle (WEP), which states that the acceleration of a body is independent of the composition of the matter involved. A WEP violation at any length scale could provide evidence that our current model of gravity is incorrect or be evidence of new forces mediated by exotic particles. Additionally, in an effort to perform a "unification" that provides a consistent framework incorporating both GR and the Standard Model, String Theory suggests that our universe may contain more than the three observed spatial dimensions. These "extra dimensions" may alter the Inverse-Square Law (ISL) at small but measureable (sub-millimeter) distance scales. Due to the weakness of Gravity compared to the other fundamental forces, laboratory tests of the WEP and ISL are difficult and require high-precision techniques. At the Humboldt State University Gravitational Physics Laboratory, we are testing the WEP and ISL at previously unexplored ranges. A planar "stepped" torsion pendulum will be used to test the WEP with unprecedented precision at the millimeter scale with multiple composition dipole configurations. Similarly, a stepped parallel-plate pendulum will be used to probe the gravitational ISL with unmatched sensitivity down to distances of 20 micrometers or less. To probe for new interactions at unprecedented levels, the system must be sensitive to torques on the order of 10^{-18} N·m. The focus of this presentation is on the current status of the laboratory and the design of an electrostatic control system which will provide means of positioning the pendulum as well as measuring any externally applied torque.

Keywords: Gravity, Inverse-square Law, Weak Equivalence Principle

1. Motivation and Background:

Although gravity was the first fundamental interaction to be described mathematically, it remains at the forefront of current physics and astronomy research. The Standard Model of quantum mechanics successfully describes all three other known fundamental interactions and the vast majority of particle physics observations, however GR, while passing all experimental tests to date, is fundamentally inconsistent with this model. The WEP is a central feature of GR, and a violation of it at any length scale would suggest that gravity may behave differently than has been assumed. Additionally, String Theory proposes the existence of additional dimensions to the three observed in order to unify the Standard Model with General Relativity. On small (sub-millimeter) distance scales, these "extra dimensions" may alter the ISL[1-2]. Furthermore, recent observations of the cosmic distance scale acceleration[3-5], attributed to Dark Energy, lead to the belief that gravity may be fundamentally different from how it is now understood.

Most alternative models of gravity predict a violation of the WEP at some level due to interactions coupled to quantities other than mass, or modifications of gravity itself[6-7]. Scalar or vector boson exchange produces forces that inherently violate the WEP over a range determined by the Compton wavelength of the exchange particle, $\lambda = h / m_b c$. The WEP has been tested with incredible precision over distance scales from 1 cm to ∞[6-7], but has never been subjected to a dedicated test in the sub-centimeter regime (corresponding to exchange boson masses greater than approximately 0.1 meV).

One of the most definitive string theory predictions in recent years suggests that gravity gets stronger at distances comparable to the size of proposed "rolled-up" extra spatial dimensions required by the theory[1]. Other models predict weakening of gravity at small scales[2]. Such scenarios are proposed in an attempt to solve the gauge hierarchy problem (the huge discrepancy in energy scales between the Planck mass and the electroweak scale).

Attempts to explain the observed cosmic distance scale acceleration (cosmological constant problem), on the other hand, have shown that data would be consistent with a theory that predicts gravity to "turn off" at distances less than about 0.1mm[8]. The observed value of the vacuum energy density, $\rho_{vac} \approx 3.8$ KeV/cm^3, corresponds to a length scale of, $R_{vac} = \sqrt[4]{\hbar c / \rho_{vac}} \approx 85\,\mu$m, which may also have fundamental significance[9].

Finally, unobserved particles predicted by string theories, such as the dilaton and moduli, may also produce new short-range forces operating through the "chameleon mechanism"[10] that could be observed in short-range tests of gravity.

2. Defining of Parameter Spaces:

2.1 ISL parameterization:

A deviation from ISL behavior is generally modeled using a Yukawa addition to the classical Newtonian potential. For point masses m_1 and m_2 separated by distance r, the potential becomes

$$V(r) = -\frac{Gm_1 m_2}{r}\left(1 + \alpha e^{-r/\lambda}\right), \tag{1}$$

where G is the Newtonian gravitational constant, α is a dimensionless scaling factor corresponding to the strength of any deviation relative to Newtonian gravity, and λ is the characteristic length scale of the deviation. Previous experiments utilizing various types of torsion pendulums have eliminated large portions of the Yukawa potential α-λ parameter space[6,11-12] (the shaded region of Figure 1 shows the current constraints in the α-λ plane). An improvement on previous experiments can be achieved by using a novel parallel-plate pendulum design and attractor drive system that is largely insensitive to ordinary Newtonian gravitational torques, while highly sensitive to short-range effects.

2.2 WEP parameterization:

It is generally assumed that a WEP violation would result in a coupling to some "charge" that is related to the seemingly conserved quantities of baryon number (atomic number, Z, plus neutron number, N), or lepton number, L ($L = Z$ for electrically neutral materials). A general scalar (-) or vector (+) Yukawa coupling would result in a potential of the form

$$V(r) = \pm\frac{1}{4\pi}\tilde{q}_1\tilde{q}_2\frac{e^{-r/\lambda}}{r}, \tag{2}$$

where $\tilde{q}_i$ are the "charges" and λ is the Compton wavelength of the exchange boson. A common parameterization assumes

$$\tilde{q} = \tilde{g}\left[Z\cos\tilde{\psi} + N\sin\tilde{\psi}\right], \tag{3}$$

where $\tilde{g}$ is a coupling constant and $\tilde{\psi}$ determines the type of charge. Recasting equation (2) in a form similar to equation (1) yields

$$V(r) = -\frac{Gm_1m_2}{r}\left(1+\tilde{\alpha}\left[\frac{\tilde{q}}{\tilde{g}\mu}\right]_1\left[\frac{\tilde{q}}{\tilde{g}\mu}\right]_2 e^{-r/\lambda}\right), \tag{4}$$

where μ is the mass of objects 1 or 2 in atomic mass units, u, and $\tilde{\alpha} = \pm\tilde{g}^2/\left(4\pi G u^2\right)$.

The WEP is well-characterized over distances from 1 cm to infinity[6-7], but it is essentially untested below the centimeter scale. While not the most sensitive to all charges, $\tilde{q}$, a combination of pendulum materials that is easy to machine is titanium and aluminum. For these materials, $(Z/\mu)_{Al}$ = 0.48181, $(N/\mu)_{Al}$ = 0.51887, $(Z/\mu)_{Ti}$ = 0.45961, and $(N/\mu)_{Ti}$ = 0.54147[6]. A copper attractor has $(Z/\mu)_a$ = 0.45636 and $(N/\mu)_a$ = 0.54475. Assuming again that the minimum measurable torque amplitude is 5×10^{-19} Nm, a peak-to-peak attractor modulation of 1 mm, and a readily attainable minimum separation s_{min} = 0.2 mm, Equation (4) can be used to find constraints in the $\tilde{\alpha}-\lambda$ parameter space. The predicted constraint on $|\tilde{\alpha}|$ for λ = 1 mm as a function of $\tilde{\psi}$ is shown in Figure 2. A minimum sensitivity in $\tilde{\alpha}$ of 10^{-4} is assumed to account for uncertainties in pendulum mass distribution, G, and other systematic errors. A vast improvement is seen over previous constraints[6] when the curves are extrapolated to the millimeter scale.

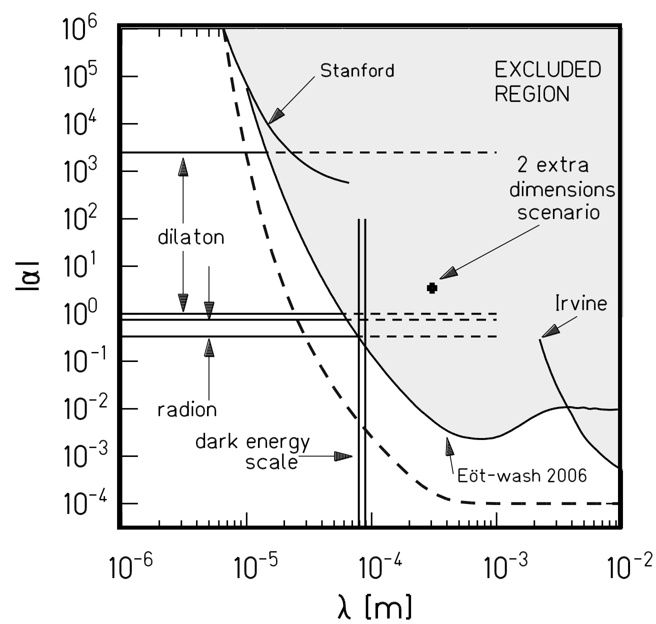

Figure 1. Current and predicted constraints on an ISL-violating interaction.

Figure 1: Current short-range experimental constraints in the α-λ Yukawa parameter space and expected sensitivity. The shaded region (to the upper right of the curves) is excluded at the 95% confidence level. Results from previous experiments are shown by the curves labeled Stanford[13], Eöt-Wash[12], and Irvine[14]. The dashed line shows the predicted sensitivity of this apparatus. Note that for some values of λ, an improvement by a factor of approximately 100 is obtained over previous efforts. Again, a minimum resolution in α of 10^{-4} is assumed to account for measurement uncertainty of the pendulum and attractor mass distributions, uncertainty in G, and other systematic errors. The dashed curve crosses α = 1 at λ = 24 μm.

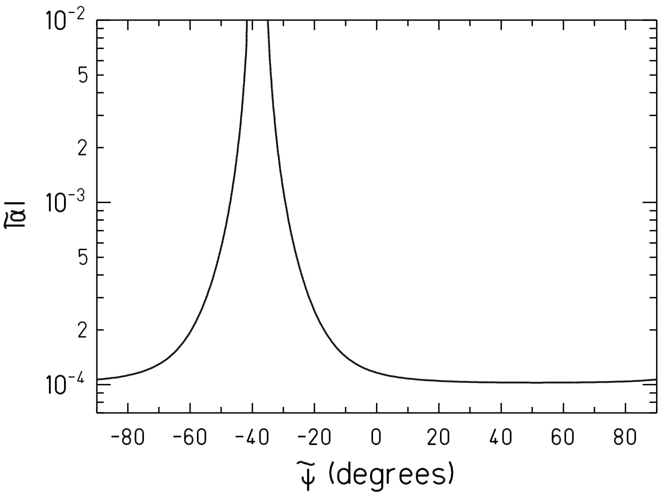

Figure 2. Predicted constraint on the strength of a WEP violating interaction.

Figure 2: Predicted constraint on the strength $|\tilde{\alpha}|$ of a WEP violating interaction as a function of the charge parameter $\tilde{\psi}$ assuming $\lambda = 1$ mm with proposed pendulum and attractor modulation parameter. The region above the curve would be tested by this experiment.

3. Apparatus and Experimental Technique:

An ideal search for short-range effects is a null experiment in which the experimental signature due to ordinary Newtonian physics is suppressed or absent, and any short-range signature is enhanced. The standard approach is to use a torsion pendulum, which naturally separates the experiment from terrestrial gravity. A null experiment may be achieved by noting that the gravitational force does not depend on distance for a test mass interacting with an infinite plane of matter. This fact can be exploited in experimental tests of gravity by using a parallel-plate configuration with planar pendulums and a comparatively large attractor plate; a simplified geometry is shown in Figure 3. The attractor-pendulum separation, *s*, is modulated by moving the attractor at angular drive frequency ω. In the ideal case of an infinite attractor plate, the Newtonian torque on the pendulum does not vary with attractor position, while any short-range interaction produces more torque on the closer, high-density "step" when *s* is smaller than the range of the interaction. This potential short-range torque modulation causes a variation of the pendulum's twist at the attractor modulation frequency. The parallel-plate design is also inherently more sensitive to short-range effects than previous "missing mass" configurations because it utilizes the full force vector instead of a single component to produce torque on the pendulum. By choosing an attractor drive frequency sufficiently different from the torsion pendulum's resonant angular frequency, ω_0, the signal will be decoupled from many external disturbances that could excite resonant motion, and the experiment will not be sensitive to the pendulum's natural mode of oscillation. However, a disadvantage of this configuration relative to previous efforts is that the signal frequency for short-range effects is the same as the attractor drive frequency, so special care is needed when investigating possible sources of systemic error.

The modulated twist of the pendulum is measured optically by reflecting laser light from the pendulum's polished surface and recording the beam deflection with an optical autocollimator. The digitized angle signal is then processed via Fourier techniques to determine the twist amplitude of the pendulum at the attractor drive frequency. The amplitude of the twist oscillation compared to the expected pendulum/attractor Newtonian and Yukawa twist amplitudes provides new limits in the α-λ parameter space, and any deviation from expected ISL (or WEP) values may be an indication of new physics.

The sensitivity of the experiment will be determined by the uncertainty in the measurement of the torque on the pendulum produced by the attractor plate, which is most importantly limited by statistical noise from the dynamics of the torsion pendulum itself and electronic noise in the autocollimator.

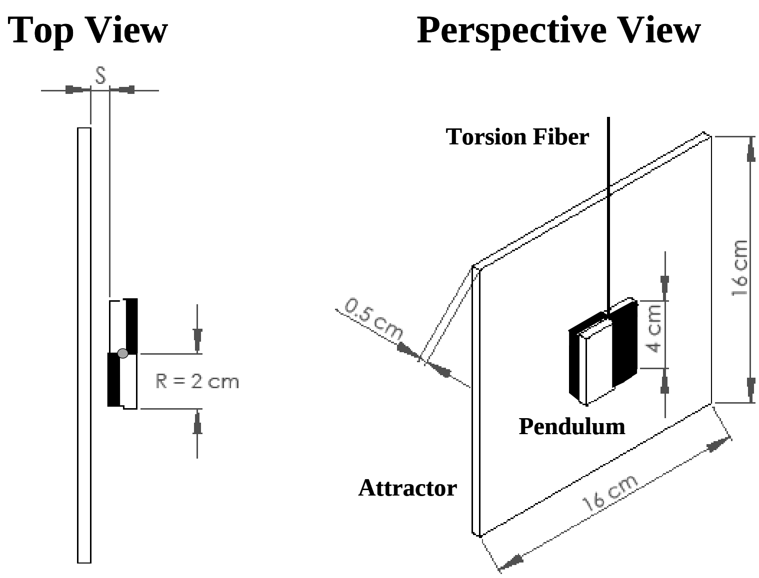

Figure 3. Basic geometry of pendulum and plate.

Figure 3: Basic geometry of a pendulum and attractor plate assumed for the calculations in this proposal. When the pendulum/attractor separation, *s*, is modulated, the pendulum's stepped design results in potentially different short-range torques applied to each side, which in turn results in a twist harmonic at a frequency equal to the chosen drive frequency for the attractor plate modulation.

4. Electrostatic Control and Sensing:

A set of electrodes placed behind the pendulum, as shown in Figure 4, will produce a torque so that the free oscillation can be reduced before data acquisition. Modeling the electrodes as each forming a parallel-plate capacitor, the applied torque on the pendulum is approximately

$$N_{applied} \approx \frac{1}{2}\frac{\partial}{\partial\theta}\left(C_1V_1^2 + C_2V_2^2\right), \tag{5}$$

where C_i and V_i are the capacitances and applied DC voltages to each electrode and θ is the pendulum twist angle with respect to some equilibrium value.

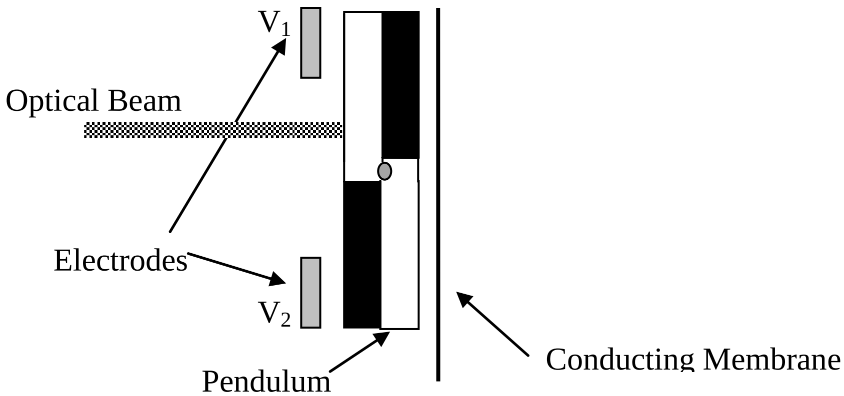

Figure 4. Configuration of pendulum and electrodes.

Figure 4: Top view of the pendulum with electrostatic actuator/sensor. A DC voltage applied to either electrode will apply a torque on the pendulum. The electrode/pendulum separation will be variable to accommodate diverse pendulum configurations. The pendulum position will be controlled with a feedback loop based on the autocollimator output. A future AC capacitance bridge implemented with the electrodes will be constructed to allow an alternate method for determining the pendulum position and providing the control signal.

By placing the electrodes a distance *d* from the pendulum and applying a voltage *V* to the electrode, the induced twist on the pendulum is found to be

$$\theta = -\frac{r\varepsilon_0 A V^2}{2\kappa d^2}, \tag{6}$$

where *r* is the distance from the center of the pendulum to the center of the electrode, *A* is the area of the interacting surfaces, κ is the spring constant of the torsion fiber and ε_0 is the permittivity of free space. Preliminary tests are being constructed using a solid bar as a mock pendulum. Figure 5 shows the theoretical range in angle over which the electrodes should be able to provide pendulum control. The plot predicts that such a system should have the ability to position the pendulum more than 2 degrees from its equilibrium position with an applied voltage of 100 Volts.

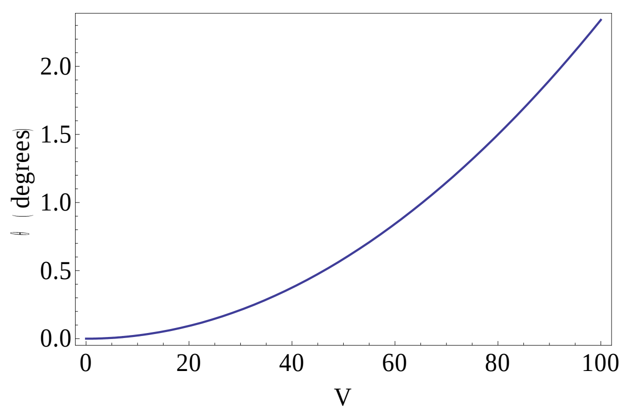

Figure 5. Predicted angular range of electrostatic control.

Figure 5: Plot of equation (6) using an area of 25 cm^2 as the interacting surface, a distance of 0.5 cm as the separation of the electrode from the torsion bar, and radius of 10 cm from the center of the bar to the center of the electrode. At this time a tungsten fiber with a rotational spring constant of $\sim 1.1\times10^{-5}$ kg m^2/s^2 is being used to support the pendulum.

Currently, a PID feedback loop is being designed for assessing the proper voltages needed to damp pendulum motion based on angular data from the autocollimator. Figure 6 shows the results of a simulation using the parameters of the electrodes and mock pendulum currently being considered.

A future improvement will be to construct a resonant capacitive bridge circuit to simultaneously sense the pendulum's position and apply damping torques. Such a system was successfully employed for a torsion pendulum designed for ground-test characterizations of noise sources for the LISA gravitational wave interferometer[15]. If the bridge circuit is as sensitive as the autocollimator it may be used to provide a secondary measure of pendulum angular deflection.

Finally, the system could be used to hold the pendulum stationary during data acquisition. The record of the applied voltages necessary to keep the pendulum stationary then provides a measure of the external torque on the pendulum and can be used in lieu of the autocollimator readout. Such a system is desirable to reduce any signal bias due to the Kuroda effect[16], and has been effectively employed in a previous experiment[17] based on angular acceleration feedback.

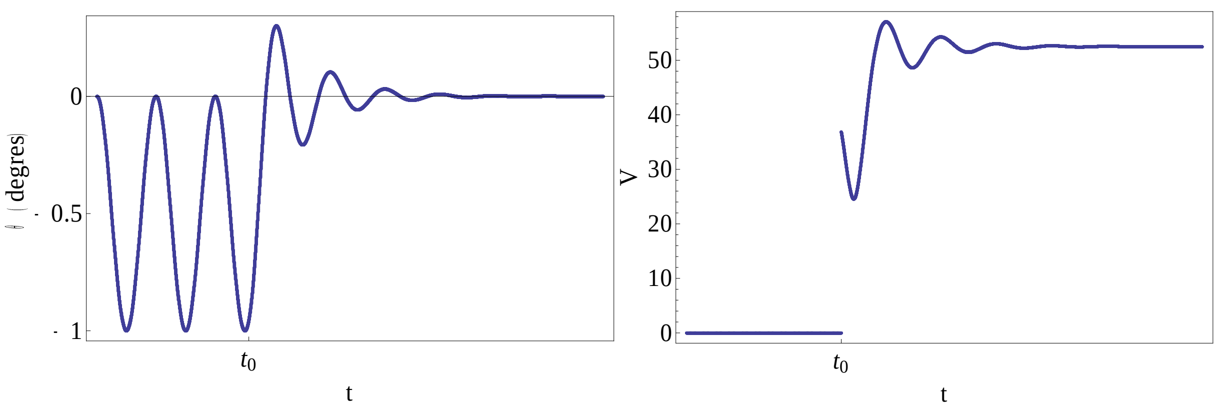

Figure 6. Simulation of PID control of a test pendulum by control electrodes.

Figure 6: (left) Predicted angular response of a torsion bar when PID control is turned on at t_0. (right) Predicted voltage needed to apply the torque calculated by the PID controller which is turned on at t_0.

5. Conclusion:

At this time a mock pendulum is being used to assess apparatus performance and characterize potential systemic error sources. The electrostatic control and sensor will provide not only means of damping and positioning of the torsion pendulum, but will yield an additional measurement of any external torque applied to the pendulum. With the design of the system finished, the parts for the electrostatic control and sensor are now in the process of being manufactured and will soon be ready for initial testing. Completion of the feedback control system and the addition of a commercial tilt sensor and a precision capacitance meter will enable measurement of pendulum/attractor separation and pendulum control system characterization. Then, after sources of noise and systemic errors have been assessed, characterized and suppressed, the apparatus will be prepared for science quality data.

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