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Experimental Tests of Gravity Below the 50-Micron Distance Scale

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Abstract

The force of gravity was the first to be mathematically described over 300 years ago; however, it remains the only fundamental force that is not currently well understood. Three other known fundamental interactions are successfully described within the Standard Model of quantum mechanics. Einstein's successful theory of gravitation, General Relativity, however, is fundamentally inconsistent with this model. The motivation to study gravity at small but measurable distances arises from the desire to probe new theories that are attempting to include gravity in a consistent framework that includes all four fundamental interactions. Such models, including versions of String Theory, suggest that our universe contains "extra dimensions," beyond the three observed spatial dimensions. At sub-millimeter distances these extra dimensions may alter the gravitational inverse-square force law. Theories involving Dark Energy, an unknown phenomenon contributing to the acceleration of our universe's expansion, also indicate that gravity may behave fundamentally differently when observed at sub-millimeter distances. Furthermore, a violation of the Weak Equivalence Principle (WEP), a central feature of General Relativity, may indicate undiscovered exotic particles predicted by String Theory. Gravity is incredibly weak compared to the other fundamental forces, making laboratory tests of the inverse-square law and WEP difficult. Such tests require high-precision techniques. At Humboldt State University, we are exploring the nature of gravity at unprecedented levels below the 50-micron distance scale. Using a novel parallel-plate torsion pendulum, we measure the pendulum's twist while an attractor mass is oscillated nearby, providing a measurable, time-varying torque on the pendulum. The size and distance dependence of this torque amplitude provide means to search for deviations from General Relativity on untested distance scales. This presentation will focus on the current status and recent results from the laboratory, including the design of a new pendulum devoted to world-leading, short-range tests of the WEP.
Keywords: Gravity, Inverse-square Law, Weak Equivalence Principle

1. Introduction and Motivation

General Relativity, while passing all experimental tests to date, is fundamentally inconsistent with the Standard Model of quantum mechanics, which has successfully described all three other known fundamental interactions and almost all observations of particle physics. String Theory attempts a "unification" of these two theories but predicts that our universe contains more than the three observed spatial dimensions. One of String Theory's predictions suggests that the strength of gravity will increase at distances comparable to the size of proposed "rolled-up" extra spatial dimensions required by the theory[1-2]. The distance scales of such alterations to the gravitational inverse-square law (ISL), although small, are potentially measureable (sub-millimeter). On the other hand, attempts to explain the observed cosmic distance scale acceleration (cosmological constant problem), attributed to an unknown phenomenon called Dark Energy, have shown that data would be consistent with a theory that predicts gravity to "turn off" at distances less than about 0.1mm[3].

A Yukawa addition to the classical Newtonian potential is generally used to model a deviation from ISL behavior[4]. For point masses m_1 and m_2 separated by a distance r, the potential becomes

$$V(r) = -\frac{Gm_1m_2}{r}\ 1+\alpha e^{-r/\lambda}\ , \tag{1}$$

where G is the Newtonian gravitational constant, α is a dimensionless scaling factor corresponding to the strength of any deviation relative to Newtonian gravity, and λ is the characteristic length scale of the deviation. Large portions of the Yukawa potential α-λ parameter space have already been eliminated by previous experiments, which have utilized a variety of different torsion pendulums. The current constraints in the α-λ parameter space are shown in Figure 1.

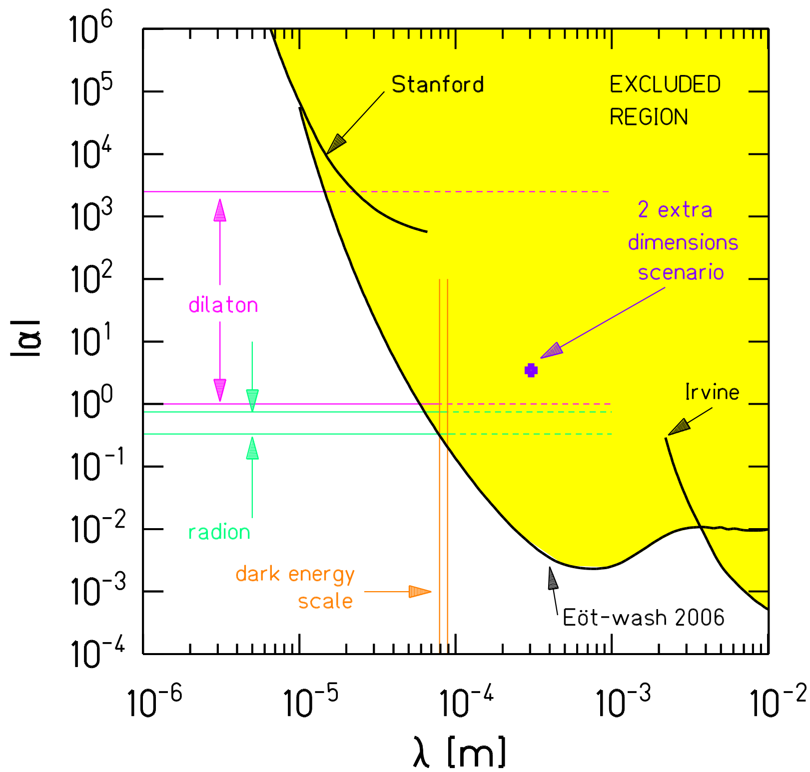

Figure 1. Existing Yukawa potential parameter constraints.

Figure 1 Current short-range experimental constraints in the α-λ parameter space. The shaded region (to the upper right of the curves) is excluded at the 95% confidence level. Results from previous experiments are shown by the curves labeled Eöt-Wash[4], Stanford[5], and Irvine[6]. Also shown are the regions where unobserved particles such as the dilaton and radion are predicted to produce short-range forces[7] as well as the Dark Energy scale.

Similarly, tests for violations of the Weak Equivalence Principle can be modeled with a potential of the form

$$V(r) = -\frac{Gm_1m_2}{r}\left(1+\tilde{\alpha}\left[\frac{\tilde{q}}{\tilde{g}\mu}\right]_1\left[\frac{\tilde{q}}{\tilde{g}\mu}\right]_2 e^{-r/\lambda}\right), \tag{2}$$

where $\tilde{q}$ is a generic charge, λ is the Compton wavelength of the exchange boson for a hypothetical interaction coupled to the generic charge, $\tilde{g}$ is a coupling constant, and μ is the mass of objects 1 or 2 in atomic mass units. A common parameterization assumes

$$\tilde{q} = \tilde{g}\left(Z\cos\tilde{\psi} + N\sin\tilde{\psi}\right), \tag{3}$$

where Z is the atomic number and N is the neutron number of the interacting materials, and $\tilde{\psi}$ is a "mixing angle" that allows the charge to be an arbitrary linear combination of Z and N. Since baryon number is the sum of the atomic number and neutron number, while the lepton number is simply the atomic number, this parameterization allows for the investigation of potential interactions that couple to the seemingly conserved quantities of baryon number and lepton number. For example, when the mixing angle is 0°, the charge will be due to the lepton number. On the other hand, when the mixing angle is equal to 45°, the charge will be proportional to baryon number. Whereas previous experiments have tested the ISL at the sub-millimeter distance scale[4, 8], the WEP has remained essentially untested below the sub-centimeter distance scale[9].

2. Testing Gravity at Humboldt State University

2.1 experimental technique

A null experiment can be used to search for short-range effects by suppressing the experimental signature due to ordinary Newtonian physics, while at the same time enhancing any short-range signature. The use of a torsion pendulum is the standard approach, which will naturally separate terrestrial gravity from short-range gravity. Since the gravitational force is not dependent on distance for a test mass interacting with an infinite plane of matter, an essentially null experiment can be achieved using a parallel-plate configuration with a stepped pendulum and a comparatively large attractor plate. Figure 2 shows such a setup.

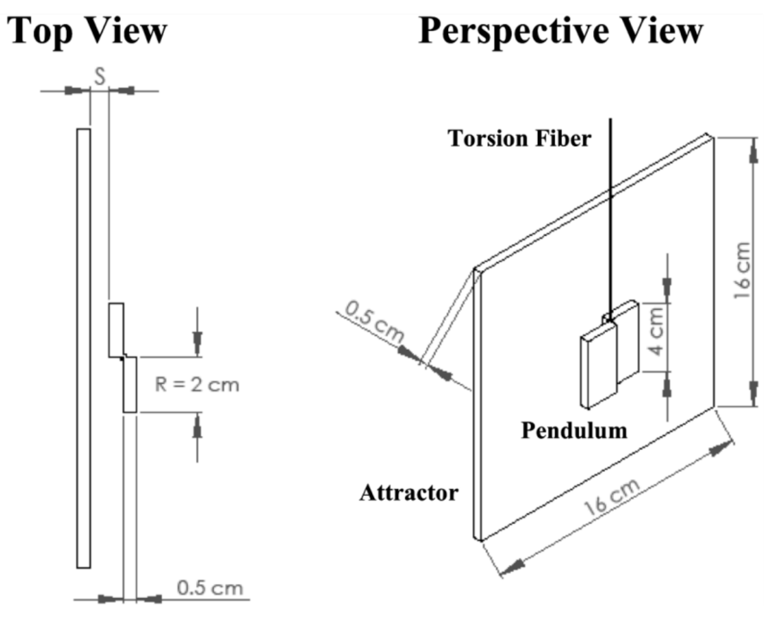

Figure 2. Basic geometry of pendulum and plate.

Figure 2 The relatively large size of the attractor plate compared to the size of the pendulum will result in a configuration which is highly insensitive to effects produced by normal, Newtonian gravity. A conducting membrane made of 20 μm-thick BeCu foil (not shown above) will be located between the attractor plate and the pendulum in order to suppress electrical forces. The pendulum, attractor, and foil will be gold-coated to minimize patch charge effects.

Moving the attractor toward and away from the pendulum at an angular drive frequency ω, different from the resonant frequency of the torsion pendulum (ω_0), will modulate the separation, s, between the attractor and the pendulum. The Newtonian torque on the system (in the ideal case of an infinite attractor plate) will not vary with attractor position, while any sub-millimeter interaction will produce a larger torque on the closer "step" of the pendulum. By reflecting a laser off the surface of the pendulum and recording the angular deflection, the oscillations of the pendulum will be measured. For an interaction with λ much smaller than the step size and pendulum/attractor dimensions, the Yukawa torque associated with the force may be approximated as[9]

$$N_Y \approx \pi\alpha G\rho_p\rho_a RA\lambda^2 e^{-s/\lambda}, \tag{4}$$

where ρ_p and ρ_a are the mass densities of the pendulum and attractor, respectively, s is the separation between the step and attractor, A is the area of the step, and R is the width of the step.

A similar composition dipole pendulum design can be used to look for interactions violating the Weak Equivalence Principle. Such a pendulum, in which the "empty" sides of the normal pendulum are filled with a different material, is shown in Figure 3.

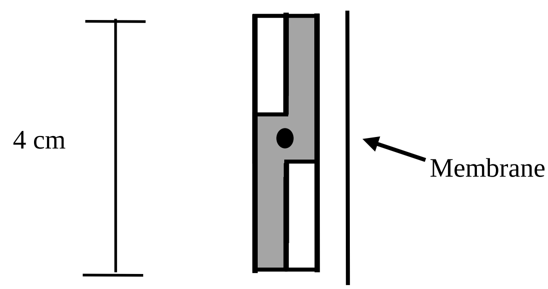

Figure 3. Top view of a composition dipole pendulum.

An attractor of density ρ_a will produce a Yukawa torque on the pendulum composed of materials with densities ρ_1 and ρ_2 that is given by[9]

$$N_{WEP} = \pi\tilde{\alpha}G\rho_A RA\lambda^2 e^{-r/\lambda}\left[\Delta Z\cos\tilde{\psi} + \Delta N\sin\tilde{\psi}\right]\left[(Z/\mu)_A\cos\tilde{\psi} + (N/\mu)_A\sin\tilde{\psi}\right], \tag{5}$$

where $\Delta Z = \rho_1 (Z/\mu)_1 - \rho_2 (Z/\mu)_2$ and $\Delta N = \rho_1 (N/\mu)_1 - \rho_2 (N/\mu)_2$.

2.2 experimental setup

In order to avoid viscous damping noise and convective thermal coupling to the pendulum, the pendulum and attractor mass are contained inside a high-vacuum chamber, which allows the experiment to be conducted with pressures as low as 10^{-6} Torr, well below the viscous regime for this pendulum design. The pendulum itself is suspended by a torsion fiber, which consists of two sections. The primary (lower) torsion fiber is a 40 cm-long, 20 μm-diameter tungsten filament with a rotational spring constant of $\kappa = 0.06$ erg/rad. The upper section is a 3 cm-long, 150 μm-diameter tungsten wire that supports an aluminum disk at the junction of the fibers. The aluminum disk resides within a magnetic field gradient, which creates a magnetic "damper" using eddy current effects to suppress the simple pendulum "swing" and the vertical "bounce" modes of oscillation.

An optical autocollimator with a pulsed laser diode is used to reflect a laser off of the pendulum and measure any deflection that occurs. The autocollimator consists of a gallium arsenide laser diode of wavelength 670 nm that is chopped by a 100 Hz, 5V square wave signal and directed through a pinhole. The light is passed through a beamsplitter and a 15 cm focal length lens collimates the divergent beam. Part of the outgoing beam is lost from the beamsplitter, reflected and absorbed in a beam dump and the remaining light is a tightly collimated beam of width ~5 mm that passes into the vacuum chamber via a transparent vacuum flange. Inside the chamber, two periscope-style mirrors, gold-coated to minimize electrostatic interactions, direct the beam to the pendulum, where it is reflected and sent back following a nearly parallel, but potentially deflected path. Upon the second pass through the beamsplitter the light is refocused to a small spot on the surface of a 4-channel position-sensitive detector (PSD). The PSD provides 4 output voltages proportional to the proximity of the laser spot to each of the four sides of the rectangular silicon surface. The four voltages are amplified and sent to the data acquisition board. The voltage difference between opposing sides gives a measure of the spot location along one axis and twist angle of pendulum. Figure 4 depicts the overall setup of the experiment.

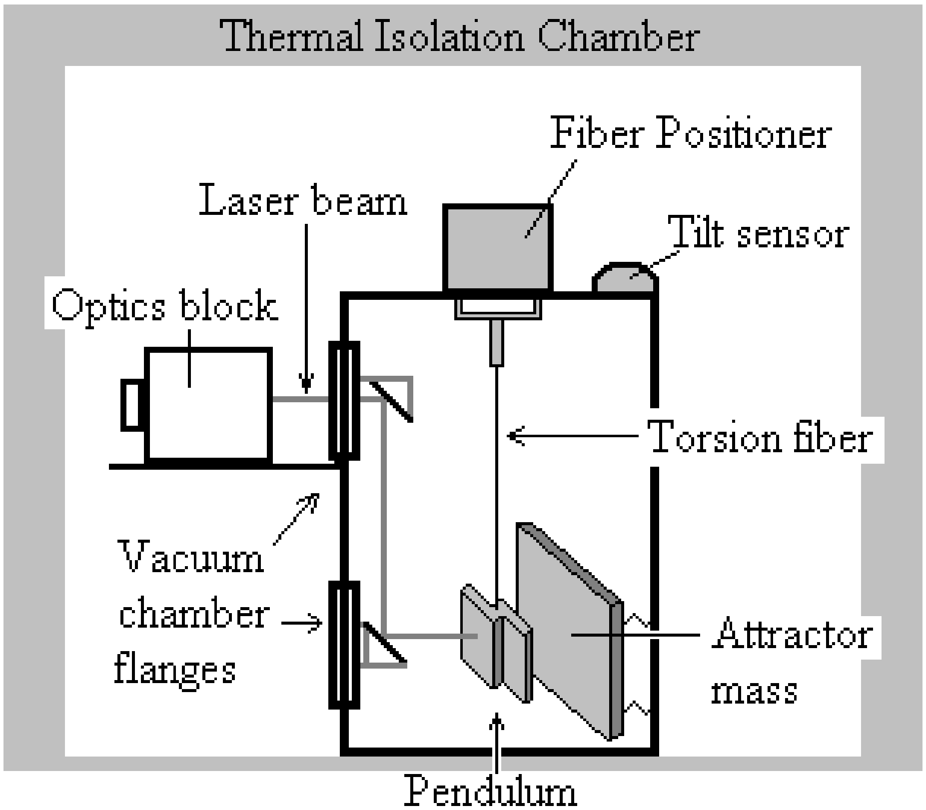

Figure 4. Overview of the experiment.

3. Expected Results

Solving for α in equation 4 gives the expected sensitivity in the α-λ parameter space for an ISL test:

$$\alpha(\lambda) = \frac{N_Y e^{s_{\min}/\lambda}}{\pi\lambda^2 G\rho_p\rho_a RA} + 0.01, \tag{6}$$

where a lower sensitivity limit of 1% is included to account for uncertainty in the calculation of the Newtonian background torque due to measurement and alignment errors. The dashed line in Figure 5 shows a plot of the sensitivity where a minimum measurable Yukawa torque of $N_Y = 5\times10^{-19}$ N·m (requiring nanoradian angular sensitivity) and a minimum attainable separation of $s_{\min} = 0.1$ mm have been assumed. This sensitivity exceeds that of previous experimental efforts by up to two orders of magnitude for certain values of λ.

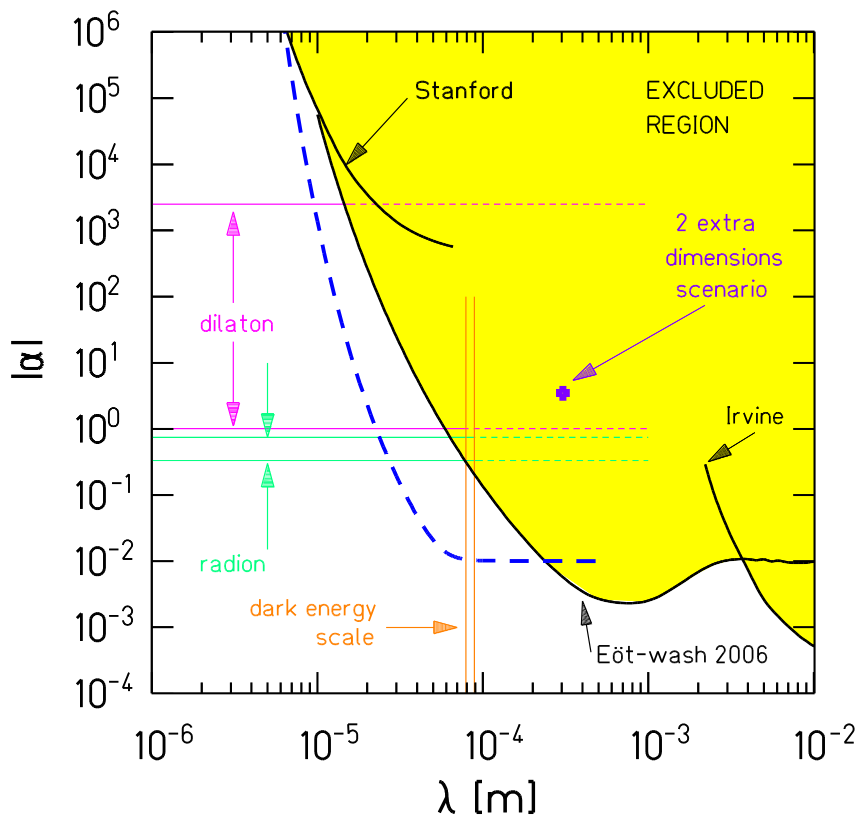

Figure 5. Current and predicted constraints on an ISL-violating interaction.

We may perform a similar analysis to obtain an expected constraint for a WEP test. Figure 6 shows the predicted constraint on the strength $|\tilde{\alpha}|$ of a WEP-violating interaction as a function of the charge parameter $\tilde{\psi}$, assuming $\lambda = 1$ mm.

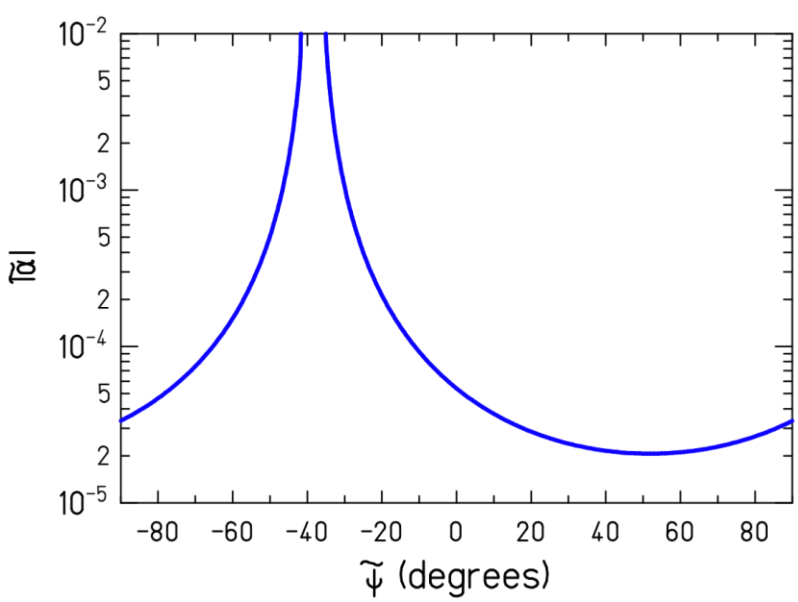

Figure 6. Predicted constraint on the strength of a WEP violating interaction for λ = 1 mm.

4. Current Results

Preliminary data has been recently collected. Using a cube with mirrors adhered to its sides as a test pendulum and with no attractor mass installed, the precision of the autocollimator can be tested to find the limits of our experiment's ability to measure changes in the angular position of the pendulum. Figure 7 shows a sample of this data. From the plot, it is evident that the pendulum's natural resonance period is about four seconds.

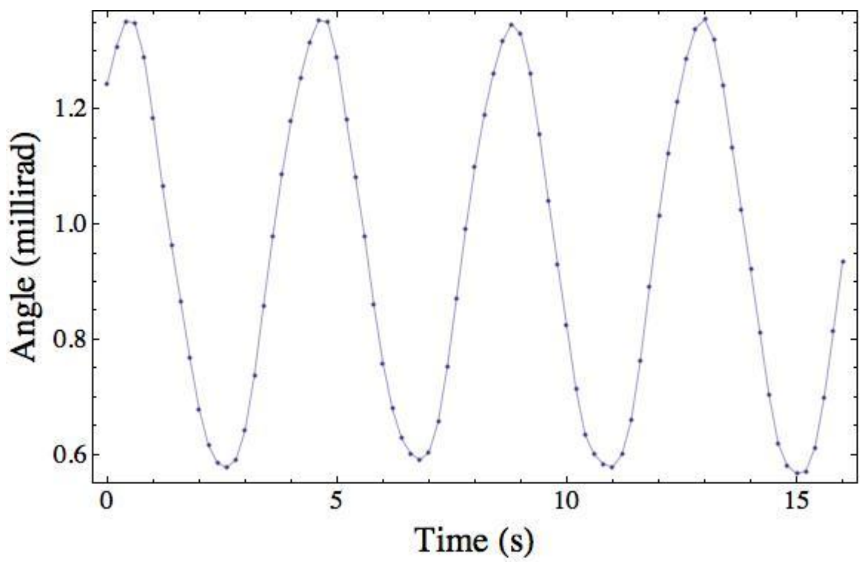

Figure 7. Sample of preliminary data.

Figure 8 shows a plot of the Fourier spectrum of the pendulum twist for a run with a length of five minutes. In agreement with the previous plot, the resonant spike in the Fourier spectrum shows the pendulum to have a natural frequency of ~ 0.25 Hz. The Fourier spectrum also shows the noise floor of the optical autocollimator. In order to obtain the required nanoradian sensitivity, this must be reduced by another order of magnitude. We are currently working to reduce this noise floor.

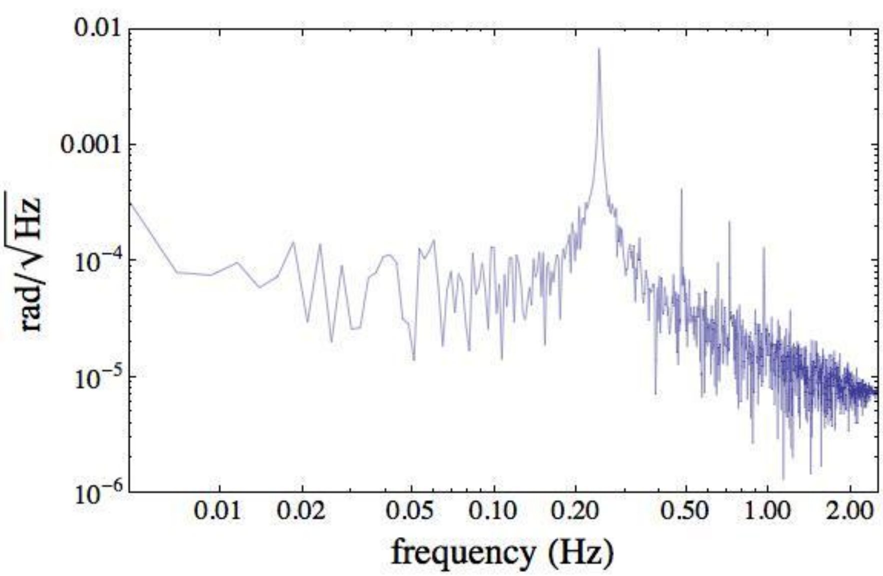

Figure 8. Fourier spectrum of the pendulum twist.

5. Conclusion

The autocollimator is currently functioning, however not yet with the necessary nanoradian angular resolution. The noise level exceeds the requirement by approximately an order of magnitude, and efforts are underway to determine the cause. Proper cable shielding and higher-quality amplifiers are currently being pursued as possible solutions. When the noise levels have been lowered sufficiently, world-leading measurements of gravity at short distances will be obtained. These measurements will allow the current constraints on ISL violations to be exceeded by up to a factor of 100. Additional torsion pendulum designs will also allow for testing of the WEP on distance scales that have not yet been investigated. Such measurements will be important for testing the predictions of String Theory and Dark Energy models that predict gravity to behave differently on small distance scales.

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